12.5 Volume of Pyramids and Cones

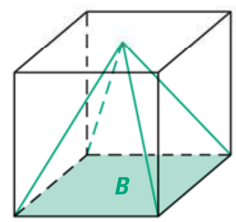
Before You found surface areas of pyramids and cones.

Now You will find volumes of pyramids and cones.

Why? So you can find the edge length of a pyramid, as in Example 2.

Key Vocabulary
- **pyramid,** *p. 810*
- **cone,** *p. 812*
- **volume,** *p. 819*

Recall that the volume of a prism is Bh, where B is the area of a base and h is the height. In the figure at the right, you can see that the volume of a pyramid must be less than the volume of a prism with the same base area and height. As suggested by the Activity on page 828, the volume of a pyramid is one third the volume of a prism.



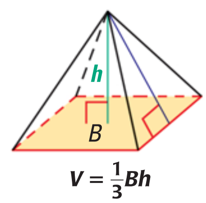
THEOREMS — For Your Notebook

THEOREM 12.9 Volume of a Pyramid

The volume V of a pyramid is

$$V = \frac{1}{3}Bh,$$

where B is the area of the base and h is the height.

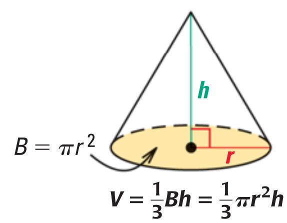


THEOREM 12.10 Volume of a Cone

The volume V of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h,$$

where B is the area of the base, h is the height, and r is the radius of the base.

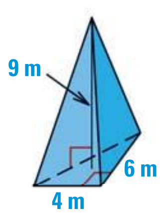


EXAMPLE 1 Find the volume of a solid

APPLY FORMULAS
The formulas given in Theorems 12.9 and 12.10 apply to right and oblique pyramids and cones. This follows from Cavalieri's Principle, stated on page 821.

Find the volume of the solid.

a.

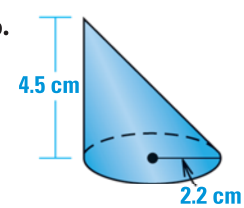


$V = \frac{1}{3}Bh$

$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9)$

$= 36 \text{ m}^3$

b.



$V = \frac{1}{3}Bh$

$= \frac{1}{3}(\pi r^2)h$

$= \frac{1}{3}(\pi \cdot 2.2^2)(4.5)$

$= 7.26\pi$

$\approx 22.81 \text{ cm}^3$

EXAMPLE 2 Use volume of a pyramid

ALGEBRA Originally, the pyramid had height 144 meters and volume 2,226,450 cubic meters. Find the side length of the square base.

Khafre's Pyramid, Egypt

Solution

$V = \frac{1}{3}Bh$ Write formula.

$2{,}226{,}450 = \frac{1}{3}(x^2)(144)$ Substitute.

$6{,}679{,}350 = 144x^2$ Multiply each side by 3.

$46{,}384 \approx x^2$ Divide each side by 144.

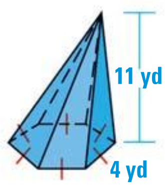
$215 \approx x$ Find the positive square root.

▸ Originally, the side length of the base was about 215 meters.

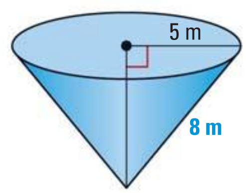
✓ GUIDED PRACTICE for Examples 1 and 2

Find the volume of the solid. Round your answer to two decimal places, if necessary.

1. Hexagonal pyramid



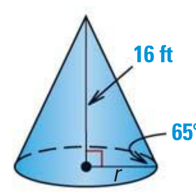
2. Right cone



3. The volume of a right cone is 1350π cubic meters and the radius is 18 meters. Find the height of the cone.

EXAMPLE 3 Use trigonometry to find the volume of a cone

Find the volume of the right cone.



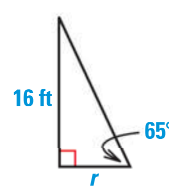
Solution

To find the radius *r* of the base, use trigonometry.

$\tan 65° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio.

$\tan 65° = \frac{16}{r}$ Substitute.

$r = \frac{16}{\tan 65°} \approx 7.46$ Solve for *r*.

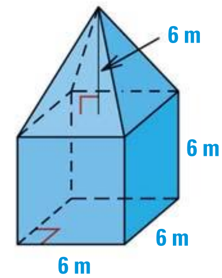


Use the formula for the volume of a cone.

$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi(7.46^2)(16) \approx 932.45 \text{ ft}^3$

EXAMPLE 4 Find volume of a composite solid

Find the volume of the solid shown.



Solution

Volume of solid	=	Volume of cube	+	Volume of pyramid

$= s^3 + \frac{1}{3}Bh$ **Write formulas.**

$= 6^3 + \frac{1}{3}(6)^2 \cdot 6$ **Substitute.**

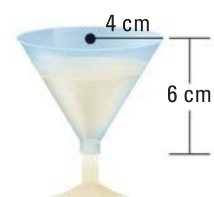
$= 216 + 72$ **Simplify.**

$= 288$ **Add.**

▶ The volume of the solid is 288 cubic meters.

EXAMPLE 5 Solve a multi-step problem

SCIENCE You are using the funnel shown to measure the coarseness of a particular type of sand. It takes 2.8 seconds for the sand to empty out of the funnel. Find the flow rate of the sand in milliliters per second. ($1 \text{ mL} = 1 \text{ cm}^3$)



Solution

STEP 1 **Find** the volume of the funnel using the formula for the volume of a cone.

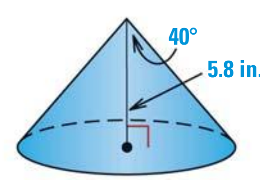
$$V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}\pi(4^2)(6) \approx 101 \text{ cm}^3 = 101 \text{ mL}$$

STEP 2 **Divide** the volume of the funnel by the time it takes the sand to empty out of the funnel.

$$\frac{101 \text{ mL}}{2.8 \text{ s}} \approx 36.07 \text{ mL/s}$$

▶ The flow rate of the sand is about 36.07 milliliters per second.

✓ GUIDED PRACTICE for Examples 3, 4, and 5



4. Find the volume of the cone at the right. Round your answer to two decimal places.

5. A right cylinder with radius 3 centimeters and height 10 centimeters has a right cone on top of it with the same base and height 5 centimeters. Find the volume of the solid. Round your answer to two decimal places.

6. **WHAT IF?** In Example 5, suppose a different type of sand is used that takes 3.2 seconds to empty out of the funnel. Find its flow rate.

12.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 17, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 18, and 35

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

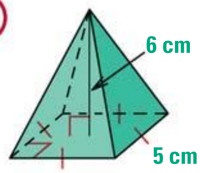
1. **VOCABULARY** *Explain* the difference between a *triangular prism* and a *triangular pyramid*. Draw an example of each.

2. ★ **WRITING** *Compare* the volume of a square pyramid to the volume of a square prism with the same base and height as the pyramid.

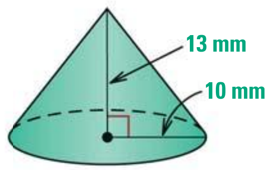
EXAMPLE 1 on p. 829 for Exs. 3–11

VOLUME OF A SOLID **Find the volume of the solid. Round your answer to two decimal places.**

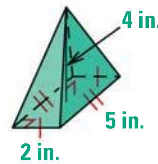
3.



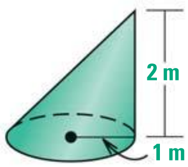
4.



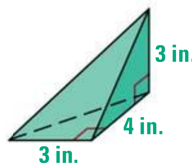
5.



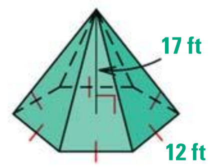
6.



7.

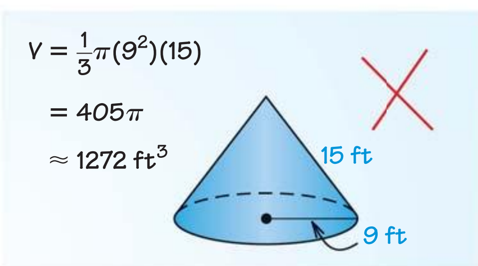


8.

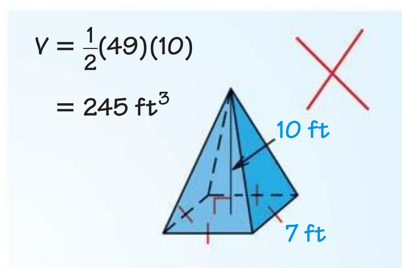


ERROR ANALYSIS *Describe* **and correct the error in finding the volume of the right cone or pyramid.**

9.



10.



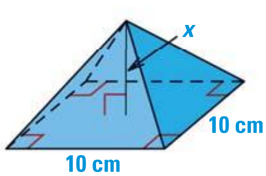
11. ★ **MULTIPLE CHOICE** The volume of a pyramid is 45 cubic feet and the height is 9 feet. What is the area of the base?

Ⓐ 3.87 ft^2 Ⓑ 5 ft^2 Ⓒ 10 ft^2 Ⓓ 15 ft^2

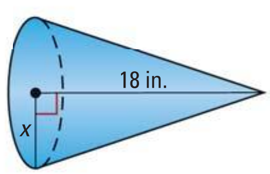
EXAMPLE 2 on p. 830 for Exs. 12–14

ALGEBRA **Find the value of x.**

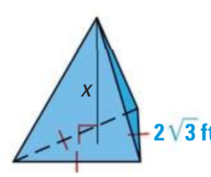
12. Volume = 200 cm^3



13. Volume = $216\pi \text{ in.}^3$



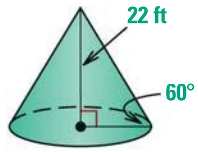
14. Volume = $7\sqrt{3} \text{ ft}^3$



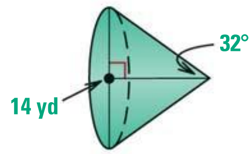
EXAMPLE 3
on p. 830
for Exs. 15–19

VOLUME OF A CONE **Find the volume of the right cone. Round your answer to two decimal places.**

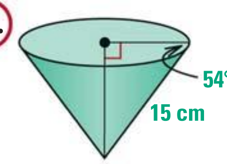
15.



16.



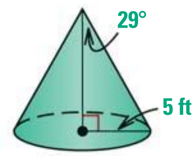
17.



18. ★ **MULTIPLE CHOICE** What is the approximate volume of the cone?

Ⓐ 47.23 ft^3 Ⓑ 236.14 ft^3

Ⓒ 269.92 ft^3 Ⓓ 354.21 ft^3

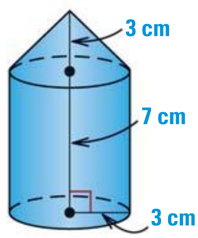


19. **HEIGHT OF A CONE** A cone with a diameter of 8 centimeters has volume 143.6 cubic centimeters. Find the height of the cone. Round your answer to two decimal places.

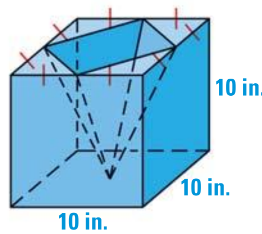
EXAMPLE 4
on p. 831
for Exs. 20–25

COMPOSITE SOLIDS **Find the volume of the solid. The prisms, pyramids, and cones are right. Round your answer to two decimal places.**

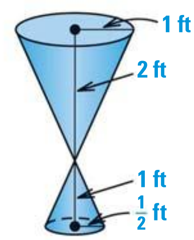
20.



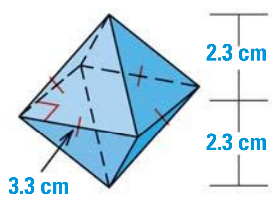
21.



22.



23.



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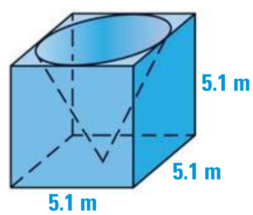
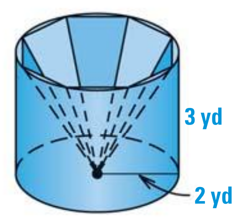
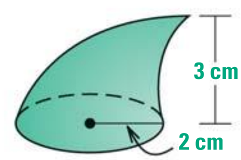


25.



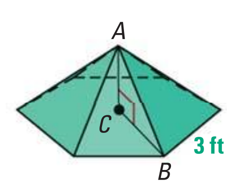
Animated Geometry at classzone.com

26. **FINDING VOLUME** The figure at the right is a cone that has been warped but whose cross sections still have the same area as a right cone with equal base area and height. Find the volume of this solid.



27. **FINDING VOLUME** Sketch a regular square pyramid with base edge length 5 meters inscribed in a cone with height 7 meters. Find the volume of the cone. *Explain* your reasoning.

28. **CHALLENGE** Find the volume of the regular hexagonal pyramid. Round your answer to the nearest hundredth of a cubic foot. In the diagram, $m\angle ABC = 35°$.



PROBLEM SOLVING

EXAMPLE 5 on p. 831 for Ex. 30

29. CAKE DECORATION A pastry bag filled with frosting has height 12 inches and radius 4 inches. A cake decorator can make 15 flowers using one bag of frosting.

a. How much frosting is in the pastry bag? Round your answer to the nearest cubic inch.

b. How many cubic inches of frosting are used to make each flower?

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POPCORN A snack stand serves a small order of popcorn in a cone-shaped cup and a large order of popcorn in a cylindrical cup.

30. Find the volume of the small cup.

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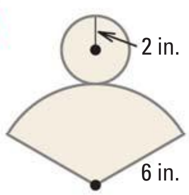
31. How many small cups of popcorn do you have to buy to equal the amount of popcorn in a large container? Do not perform any calculations. *Explain*.

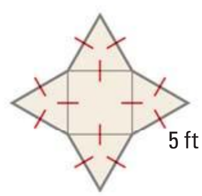
32. Which container gives you more popcorn for your money? *Explain*.

USING NETS In Exercises 33 and 34, use the net to sketch the solid. Then find the volume of the solid. Round your answer to two decimal places.

33.



34.



35. ★ **EXTENDED RESPONSE** A pyramid has height 10 feet and a square base with side length 7 feet.

a. How does the volume of the pyramid change if the base stays the same and the height is doubled?

b. How does the volume of the pyramid change if the height stays the same and the side length of the base is doubled?

c. *Explain* why your answers to parts (a) and (b) are true for any height and side length.

36. AUTOMATIC FEEDER Assume the automatic pet feeder is a right cylinder on top of a right cone of the same radius. (1 cup = 14.4 in.^{3})

a. Calculate the amount of food in cups that can be placed in the feeder.

b. A cat eats one third of a cup of food, twice per day. How many days will the feeder have food without refilling it?

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

37. NAUTICAL PRISMS The nautical deck prism shown is composed of the following three solids: a regular hexagonal prism with edge length 3.5 inches and height 1.5 inches, a regular hexagonal prism with edge length 3.25 inches and height 0.25 inch, and a regular hexagonal pyramid with edge length 3 inches and height 3 inches. Find the volume of the deck prism.

38. MULTI-STEP PROBLEM Calculus can be used to show that the average value of r^2 of a circular cross section of a cone is $\frac{r_b^2}{3}$, where r_b is the radius of the base.

a. Find the average area of a circular cross section of a cone whose base has radius R.

b. Show that the volume of the cone can be expressed as follows:

V_{cone} = (Average area of a circular cross section) • (Height of cone)

39. MULTIPLE REPRESENTATIONS Water flows into a reservoir shaped like a right cone at the rate of 1.8 cubic meters per minute. The height and diameter of the reservoir are equal.

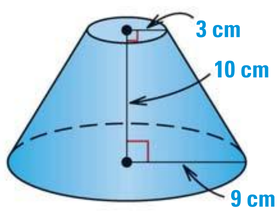
a. Using Algebra As the water flows into the reservoir, the relationship $h = 2r$ is always true. Using this fact, show that $V = \frac{\pi h^3}{12}$.

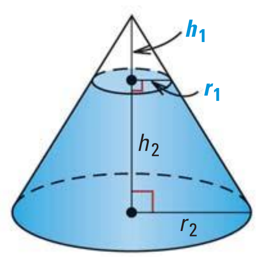
b. Making a Table Make a table that gives the height h of the water after 1, 2, 3, 4, and 5 minutes.

c. Drawing a Graph Make a graph of height versus time. Is there a linear relationship between the height of the water and time? *Explain.*

FRUSTUM A frustum of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Use the information to complete Exercises 40 and 41.

One method for calculating the volume of a frustum is to add the areas of the two bases to their geometric mean, then multiply the result by $\frac{1}{3}$ the height.





40. Use the measurements in the diagram at the left above to calculate the volume of the frustum.

41. Complete parts (a) and (b) below to write a formula for the volume of a frustum that has bases with radii r_1 and r_2 and a height h_2.

a. Use similar triangles to find the value of h_1 in terms of h_2, r_1, and r_2.

b. Write a formula in terms of h_2, r_1, and r_2 for V_{frustum} = (Original volume) − (Removed volume).

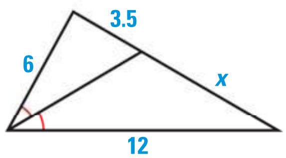
c. Show that your formula in part (b) is equivalent to the formula involving geometric mean described above.

42. **CHALLENGE** A square pyramid is inscribed in a right cylinder so that the base of the pyramid is on a base of the cylinder, and the vertex of the pyramid is on the other base of the cylinder. The cylinder has radius 6 feet and height 12 feet. Find the volume of the pyramid. Round your answer to two decimal places.

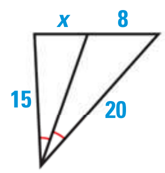
MIXED REVIEW

In Exercises 43–45, find the value of *x*. *(p. 397)*

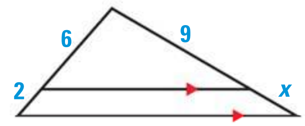
43.



44.

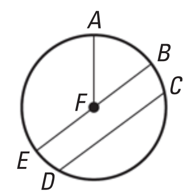


45.



PREVIEW
Prepare for Lesson 12.6 in Exs. 46–52.

46. Copy the diagram at the right. Name a radius, diameter, and chord. *(p. 651)*



47. Name a minor arc of $\odot F$. *(p. 659)*

48. Name a major arc of $\odot F$. *(p. 659)*

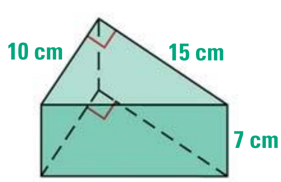
Find the area of the circle with the given radius *r*, diameter *d*, or circumference *C*. *(p. 755)*

49. $r = 3$ m **50.** $d = 7$ mi **51.** $r = 0.4$ cm **52.** $C = 8\pi$ in.

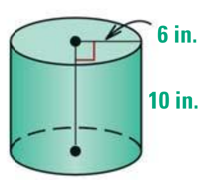
QUIZ *for Lessons 12.4–12.5*

Find the volume of the figure. Round your answer to two decimal places, if necessary. *(pp. 819, 829)*

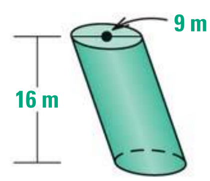
1.



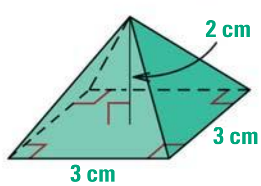
2.



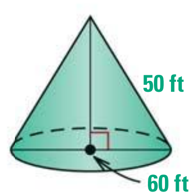
3.



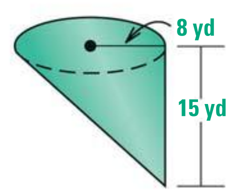
4.



5.



6.



7. Suppose you fill up a cone-shaped cup with water. You then pour the water into a cylindrical cup with the same radius. Both cups have a height of 6 inches. Without doing any calculation, determine how high the water level will be in the cylindrical cup once all of the water is poured into it. *Explain* your reasoning. *(p. 829)*